

2.

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 1 & -1 & h & 3 \end{array} \right] \xrightarrow{-2 \cdot R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & -1 & h & 3 \end{array} \right] \\ & \xrightarrow{-R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -1 \\ 0 & -3 & h-1 & 2 \end{array} \right] \xrightarrow{-R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & h-2 & 3 \end{array} \right] \end{aligned}$$

So, the system is consistent when $h - 2 \neq 0 \iff \boxed{h \neq 2}$.

3.

To determine if the system $Ax = b$ is consistent for every $b \in \mathbb{R}^3$ we need to determine if the columns of A are linearly independent or the determinant is nonzero.

$$\det(A) = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = (-2)^3 + 1 + 1 - (-2) - (-2) - (-2) = -8 + 2 + 2 + 2 + 2 = 0$$

So, the system is not consistent for every vector b .