

Given a continuous function $f(x, y)$, the tangent slope at a point P in the direction \vec{u} is given as the directional derivative of the surface in the direction of the vector \vec{u} .

The directional derivative of a function at a given point P in a direction of a unit vector is the inclination of the surface if you are to walk in the direction of \vec{u} standing at the point P .

So, the tangent slope in the direction of \vec{u} is

$$D_{\vec{u}}f(x_P, y_P) = \nabla f(x_P, y_P) \cdot \frac{\vec{u}}{|\vec{u}|} = a.$$

Therefore the tangent slope in the opposite direction of \vec{u} which is $-\vec{u}$ is

$$D_{-\vec{u}}f(x_P, y_P) = \nabla f(x_P, y_P) \cdot \left(-\frac{\vec{u}}{|\vec{u}|}\right) = -\nabla f(x_P, y_P) \cdot \frac{\vec{u}}{|\vec{u}|} = \boxed{-a}.$$

P.S. It is needed that the surface is continuous, otherwise we cannot take derivatives when we compute the gradient vector $\nabla f(x, y)$.