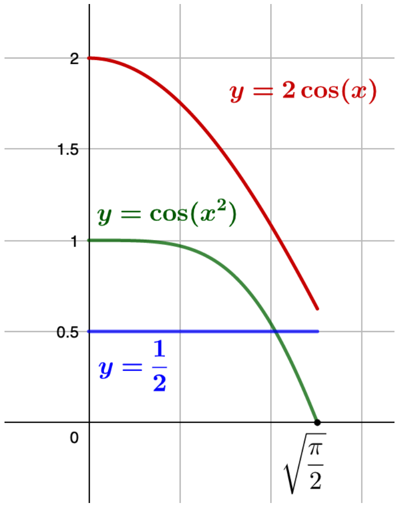
The graph of $\cos(x^2)$ is bounded above by $y = 2\cos(x)$ on $\left[0, \sqrt{\frac{\pi}{2}}\right]$,



therefore the area under the graphs are in the following relationship:

$$\int_0^{\sqrt{\pi/2}} \cos(t^2)\ dt \le \int_0^{\pi/2} 2\cos(t)\ dt = 2\sin(t)\Big|_0^{\sqrt{\pi/2}} = 2\sin\left(\sqrt{\frac{\pi}{2}}\right).$$

Also, the area below the green line is more than the area below the horizontal line $y = \frac{1}{2}$, therefore,

$$\int_0^{\sqrt{\pi/2}} \cos(t^2)\ dt \ge \int_0^{\sqrt{\pi/2}} \frac{1}{2}\ dt = \frac{\sqrt{\pi}}{2\sqrt{2}}.$$

So, we proved the inequality: $\frac{\sqrt{\pi}}{2\sqrt{2}} \le \int_0^{\sqrt{\pi/2}} \cos(t^2)\ dt \le 2\sin\left(\sqrt{\frac{\pi}{2}}\right)$.